points (Fig. 2) is probably due to the limitation of using mean local velocity rather than the convection velocity of the turbulent eddies[8] in estimating the length scale using Taylor's hypothesis. The scale of the eddies increases at a very slow rate with radial position

$$1(x) = 0.09(y/r_0) + 0.172 \quad (4)$$

across the considerable portion of pipe section. This rate deviates to a much larger value beyond $y/r_0 > 0.85$. It is unfortunate that the measurements are not extended to the so-called viscous layers. The trend, however, shows that the eddies are elongated near the wall, which is in agreement with the postulation of Townsend[10] and measurements of others.[4,5]

Discussion

The results indicate the dominance of large-scale motions in the wall region of the pipe. As the eddies are stretched more and more due to the presence of the wall, the skewness and flatness factors of the probability distributions tend to increase to high values compared to the Gaussian distributions. The convection velocities of these large-scale motions are also very high compared to the mean local velocity.[4] This is probably a result of the prominance of the three-dimensional disturbances in the wall region.

Earlier investigators indicated resemblance between the turbulent characteristics in the wall region of a pipe flow and that of boundary layer, though the physical constraints of the two flows are different. In boundary-layer flow, one finds a sharp interface between turbulent and nonturbulent regions that implies intermittency, whereas a developed pipe flow is characterized by the absence of intermittency. A significant deviation in the characteristics of the turbulent signals between these two cases, which are logically related to the physics of the flow, is also noticed. It appears that the detailed structure and mechanism of the motion in these two flows would be different. Thus, it is not fully understood that Kline's model of turbulent structure in the boundary layer may be strictly postulated in the pipe flow. A further study to this end seems to be useful.

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Some Characteristics of Laminated Filamentary Composites

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In a recent design of a certain component of an advanced aircraft, laminated composite material was used for the cover skin. The material was arranged in such a way that 40% of the filaments lies in the direction of a reference axis. The other 60% lies in the ±45° directions symmetric to the reference axis. The material is macroscopically orthotropic with its major principal axis oriented in the direction of the reference axis for maximum efficiency. It was rather surprising to observe that the direction of maximum Young's modulus does not coincide with that of the major principal axis. This note is to review the mechanics of the orthotropic materials and to determine the relationship between the independent variables, namely, E_1, E_2, G_{12}, ν_{12} and θ (for notations see Ref. 1) which would cause the seemingly improbable phenomenon to occur.

The basic stress-strain relationship of the orthotropic material is

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \dfrac{\gamma_{12}}{2} \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{E_1} & -\dfrac{\nu_{21}}{E_2} & 0 \\ -\dfrac{\nu_{12}}{E_1} & \dfrac{1}{E_2} & 0 \\ 0 & 0 & \dfrac{1}{2G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_2 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \quad (1)$$

or

$$\epsilon_{12} = S_{12}\sigma_{12} \quad (2)$$

where the subscripts 1 and 2 indicate the directions of the major and minor principal axes. For other mutually perpendicular directions x and y which do not coincide with directions 1 and 2, the stress-strain relationships are

$$\epsilon_{xy} = S_{xy}\sigma_{xy} \quad (3)$$

and S_{xy} may be obtained by following transformation

$$S_{xy} = T^{-1}S_{12}T \quad (4)$$

where

$$T = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \sin 2\theta \\ \sin^2\theta & \cos^2\theta & -\sin 2\theta \\ -\frac{1}{2}\sin 2\theta & \frac{1}{2}\sin 2\theta & \cos 2\theta \end{bmatrix} \quad (5)$$

Equation (3) may be expressed in the following manner:

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \dfrac{\gamma_{xy}}{2} \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{E_x} & -\dfrac{\nu_{yx}}{E_y} & -\dfrac{m_x}{E_1} \\ -\dfrac{\nu_{xy}}{E_x} & \dfrac{1}{E_y} & -\dfrac{m_y}{E_1} \\ -\dfrac{m_x}{E_1} & -\dfrac{m_y}{E_1} & \dfrac{1}{2G_{xy}} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad (6)$$

Since we are interested in the relationship between E_x and the independent variables: E_1, E_2, G_{12} and ν_{12}, the expansion of Eq. (4) yields

$$\frac{E_1}{E_x} = \cos^4\theta + \frac{E_1}{E_2}\sin^4\theta + \frac{1}{4}\left(\frac{E_1}{G_{12}} - 2\nu_{12}\right)\sin^2 2\theta \quad (7)$$

Received October 4, 1970; revision received March 9, 1971.

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or

$$\frac{E_1}{E_x} = \cos^4\theta + a \sin^4\theta + b \sin^2 2\theta \quad (8)$$

with

$$a = E_1/E_2, \quad \text{and} \quad b = \frac{1}{4}(E_1/G_{12} - 2\nu_{12}) \quad (9)$$

Equation (8) may be written as

$$\frac{E_1}{E_x} = (1 + a - 4b) \cos^4\theta + (4b - 2a) \cos^2\theta + a \quad (10)$$

Differentiating E_1/E_x with respect to θ, and setting $d(E_1/E_x)/d\theta$ to zero, we have

$$\frac{d(E_1/E_x)}{d\theta} = -4(1 + a - b) \cos\theta \sin\theta - 2(4b - 2a) \sin\theta \cos\theta = 0 \quad (11)$$

$$= \sin\theta \cos\theta[4(4b - a - 1) \cos^2\theta - 2(4b - 2a)] = 0 \quad (12)$$

Equation (12) indicates that there are relative maximum and minimum values of E_1/E_x at $\theta = \pi/2$ and $\theta = 0$, which are obvious. In addition there may exist an absolute maximum or minimum value of E_1/E_x at

$$\theta = \cos^{-1}[(2b - a)/(4b - a - 1)]^{1/2} \quad (13)$$

For $0 < \theta < \pi/2$, either of the following conditions must be met

$$4b - a - 1 > 2b - a > 0 \quad (14)$$

or

$$4b - a - 1 < 2b - a < 0 \quad (15)$$

If we substitute Eq. (9) back into the inequalities (14) and (15), we have the following criteria:

$$G_{12} < \frac{E_1}{2(E_1/E_2 + \nu_{12})} \quad \text{or} \quad G_{12} > \frac{E_1}{2(1 + \nu_{12})} \quad (16)$$

It is interesting to note that inequalities (16) bear some imilarity to the well-known $G = E/[2(1 + \nu)]$ for isotropic materials. It can be further proved that if $G_{12} < E_1/[2(E_1/E_2 + \nu_{12})]$, an absolute minimum value E_x which is less than E_2 exists, on the other hand if $G_{12} > E_1/[2(1 + \nu_{12})]$, an absolute maximum value of E_x which is greater than E_1 exists. If G_{12} is within the range of $E_1/[2(E_1/E_2 + \nu_{12})]$ and $E_1/[2(1 + \nu_{12})]$, then E_1 and E_2 represent the absolute maximum and minimum values of Young's modulus of the orthotropic material.

The composite material described in the beginning of this note has a shear modulus, $G_{12} > E_1/[2(1 + \nu_{12})]$, therefore, an absolute maximum value of E_x exists. Further examples were also found in the literature. In Ref. 2 the example shows the material of which the shear modulus falls between $E_1/[2(1 + \nu_{12})]$ and $E_1/[2(E_1/E_2 + \nu_{12})]$; Fig. 1.5 (p. 10) indicates that E_1 and E_2 are the absolute maximum and minimum values of E_x. In Ref. 3, the figure (p. 57) shows the variation of E_x vs θ of a boron-epoxy lamina, and indicates the existence of an absolute minimum value less than E_2. Even though the elastic moduli of the material were not given in that reference, it is conceivable that for a single lamina of boron composite the shear modulus G_{12} is very low, and consequently $G_{12} < E_1/[2(E_1/E_2 + \nu_{12})]$ is satisfied.

In Ref. 4 in which anisotropic strength of composites was investigated, it was reported that a minimum normal strength exists between $\theta = 0$ and $\pi/2$ when $T^2 \ll X^2$ and $Y > (2T)^{1/2}$. This phenomenon is similar to the variation of Young's modulus with respect to θ as reported in the present note. It is also noteworthy that there may exist an absolute maximum normal strength between $\theta = 0$ and $\pi/2$ when $T > X$, and this happens when the composite material is laminated in such a way that the shear strength is greater than the normal strength in the major principal direction.

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A New Oxidizer for Auxiliary Ignition in Liquid Rocket Motors

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Introduction

Kinetic and mechanistic studies of combustion reactions are sometimes helpful in locating important steps which govern the combustion processes. A correct knowledge of the combustion mechanism can suggest various possible modifications which could be made in order to improve the performance of a particular system. Very few such studies have been carried out[1–5]. Rastogi and co-workers have intensively studied the kinetics and mechanism of the aniline and RFNA system[1–3,5]. The mechanistic study shows that both oxidation and nitration of aniline take place during combustion. Thus the combustion of aniline can be facilitated either by accelerating the oxidation route or the nitration route or both. The present studies were undertaken to discover new systems which could enhance the ignition process. The (RFNA + SO_2)/aniline system was investigated since the oxidizer system promotes quicker nitration[6].

Experimental

Preparation of oxidizer

The oxidizer system was prepared by bubbling SO_2 gas into RFNA kept in an ice bath for approximately 5–6 hours. The density of (RFNA + SO_2) mixture and RFNA was 1.6877 and 1.4032 respectively at 20°C. Aniline was purified by twice distilling it over zinc dust with the aid of a fractionating column.

Measurement of ignition delay

Ignition delay for the new system was measured by the oscilloscope method described earlier.[7] The results are given in Table 1. The uncertainty in the measurement was of the order of ±.05 sec. However, the ignition delay for RFNA-aniline system was measured by cup-test method.

Table 1 gives the ignition delay of combustion of aniline with (RFNA + SO_2) for various values of oxidizer-fuel ratio.

Received December 4, 1970; revision received February 23, 1971.

Thanks are due to Ministry of Defence, India, for the grant of Research fellowship to H. J. Singh.

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